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R Notebook

library(MASS)  
library(MVN)  
library(biotools)

## ---  
## biotools version 4.2

library(readxl)  
library(ggplot2)  
library(tidyverse)

data13 <- read\_excel("C:/Users/lili/Downloads/data13.xlsx",sheet="Sheet2")  
knitr::kable(head(data13, 10))

| GPA | GMAT | admit |
| --- | --- | --- |
| 2.96 | 596 | 1 |
| 3.14 | 473 | 1 |
| 3.22 | 482 | 1 |
| 3.29 | 527 | 1 |
| 3.69 | 505 | 1 |
| 3.46 | 693 | 1 |
| 3.03 | 626 | 1 |
| 3.19 | 663 | 1 |
| 3.63 | 447 | 1 |
| 3.59 | 588 | 1 |

data13$admit <- as.factor(data13$admit)  
summary(data13)

## GPA GMAT admit   
## Min. :2.130 Min. :313.0 1:31   
## 1st Qu.:2.600 1st Qu.:425.0 2:28   
## Median :3.010 Median :482.0 3:26   
## Mean :2.975 Mean :488.4   
## 3rd Qu.:3.300 3rd Qu.:538.0   
## Max. :3.800 Max. :693.0

mnv <- manova(cbind(GPA,GMAT)~admit, data=data13)  
summary(mnv)

## Df Pillai approx F num Df den Df Pr(>F)   
## admit 2 1.0096 41.797 4 164 < 2.2e-16 \*\*\*  
## Residuals 82   
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Terlihat bahwa tolak H0 sehingga ada perbedaan yang signifikan

**Uji Homogenitas**

H0 : ∑1=∑2=∑3

H1 : Minimal ada satu yang berbeda

boxM(data13[,-3],data13$admit)

##   
## Box's M-test for Homogeneity of Covariance Matrices  
##   
## data: data13[, -3]  
## Chi-Sq (approx.) = 16.074, df = 6, p-value = 0.01336

Terlihat bahwa tidak memenuhi asumsi homogenitas (tolak H0). Namun pada tutorial kali ini akan dianggap memenuhi asumsi homogenitas.

**#Uji Normalitas**

mvn(data13[, -3])

## $multivariateNormality  
## Test HZ p value MVN  
## 1 Henze-Zirkler 1.136135 0.01526607 NO  
##   
## $univariateNormality  
## Test Variable Statistic p value Normality  
## 1 Anderson-Darling GPA 0.5759 0.1307 YES   
## 2 Anderson-Darling GMAT 0.4648 0.2483 YES   
##   
## $Descriptives  
## n Mean Std.Dev Median Min Max 25th 75th Skew  
## GPA 85 2.974588 0.4289954 3.01 2.13 3.8 2.6 3.3 -0.04826749  
## GMAT 85 488.447059 81.5223466 482.00 313.00 693.0 425.0 538.0 0.38849916  
## Kurtosis  
## GPA -1.0091148  
## GMAT -0.1362411

Terlihat bahwa data semua variabel berdistribusi normal

set.seed(123)  
k <- length(levels(data13$admit))  
n <- nrow(data13)  
# data training sebanyak  
idx <- sample(1:n, 36)  
data13.tr <- data13[idx,] #data training  
data13.ts <- data13[-idx,] #data testing  
# Analisis Diskriminan  
fit<-lda(admit~.,data=data13.tr)  
# Output dari analisis diskriminan  
fit

## Call:  
## lda(admit ~ ., data = data13.tr)  
##   
## Prior probabilities of groups:  
## 1 2 3   
## 0.2777778 0.3611111 0.3611111   
##   
## Group means:  
## GPA GMAT  
## 1 3.428000 585.4000  
## 2 2.515385 461.5385  
## 3 2.923846 457.8462  
##   
## Coefficients of linear discriminants:  
## LD1 LD2  
## GPA -5.861113047 2.2030403  
## GMAT -0.009791428 -0.0154151  
##   
## Proportion of trace:  
## LD1 LD2   
## 0.9724 0.0276

**Prior : Jumlah sampel yg masuk**

Jumlah sampel yang masuk 28% dari group 1, 36% dari group 2, 36% dari group 3

**Means : rata-rata tiap kelompok.**

Rata-rata GPA group1 adalah 3.4, group2 adalah 2.5, group3 adalah 2.9 Rata-rata GMAT group1 adalah 585.4, group2 adalah 461.53, group3 adalah 457.84

**koefisien diskriminannya**

LD1 : -5.86GPA - 0.0098GMAT

LD2 = 2.2GPA - 0.015GMAT

**proporsi of trace : proporsi menjelaskan**

LD1 menjelaskan 97% dari data, sedangkan LD2 menjelaskan 3% dari data

zscore <- predict(fit,data13[,-3])  
knitr::kable(head(zscore$x, 10))

| LD1 | LD2 |
| --- | --- |
| -1.248352 | -1.4668431 |
| -1.099006 | 0.8257618 |
| -1.656018 | 0.8632691 |
| -2.506910 | 0.3238023 |
| -4.635944 | 1.5441507 |
| -5.128677 | -1.8605879 |
| -1.952372 | -1.7750833 |
| -3.252433 | -1.9929557 |
| -3.716375 | 2.3060442 |
| -4.862521 | 0.0443931 |

kor1 <- t(cor(zscore$x[,1],data13[,-3]))  
kor2 <- t(cor(zscore$x[,2],data13[,-3]))  
tabkor<-data.frame(kor1,kor2)  
tabkor

## kor1 kor2  
## GPA -0.9710885 0.3119494  
## GMAT -0.6592011 -0.6996044

pred <- predict(fit,data13.ts[,-3])  
pred

## $class  
## [1] 3 3 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 3 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 2 3 3 3 3  
## [39] 3 3 1 3 3 3 3 3 3 3 3  
## Levels: 1 2 3  
##   
## $posterior  
## 1 2 3  
## 1 2.423080e-01 5.248747e-03 7.524432e-01  
## 2 2.357844e-02 1.079055e-03 9.753425e-01  
## 3 1.957225e-01 2.307773e-04 8.040467e-01  
## 4 9.324878e-01 4.366146e-06 6.750786e-02  
## 5 9.999733e-01 3.545382e-12 2.667887e-05  
## 6 9.999998e-01 1.973107e-13 1.825904e-07  
## 7 9.995659e-01 4.441686e-08 4.340786e-04  
## 8 9.919158e-01 3.302344e-07 8.083831e-03  
## 9 9.985600e-01 1.300292e-08 1.439965e-03  
## 10 9.999472e-01 8.268495e-11 5.275590e-05  
## 11 9.990919e-01 2.824614e-09 9.080749e-04  
## 12 9.996905e-01 9.242793e-10 3.095306e-04  
## 13 9.736707e-01 6.941992e-07 2.632865e-02  
## 14 9.968919e-01 3.577356e-08 3.108048e-03  
## 15 8.984928e-01 7.959115e-06 1.014992e-01  
## 16 7.247398e-01 6.207388e-05 2.751981e-01  
## 17 9.999868e-01 6.184420e-12 1.319947e-05  
## 18 4.777424e-01 1.304243e-05 5.222446e-01  
## 19 9.979445e-01 2.847089e-08 2.055518e-03  
## 20 9.999991e-01 1.845609e-14 9.151377e-07  
## 21 1.000000e+00 3.427203e-17 2.548323e-09  
## 22 1.264447e-12 9.926243e-01 7.375656e-03  
## 23 3.759979e-15 9.997133e-01 2.866532e-04  
## 24 6.493805e-11 9.941372e-01 5.862825e-03  
## 25 1.750313e-14 9.982297e-01 1.770348e-03  
## 26 2.150635e-10 9.661961e-01 3.380387e-02  
## 27 1.661307e-14 9.980421e-01 1.957947e-03  
## 28 7.381813e-09 9.610313e-01 3.896867e-02  
## 29 3.064541e-07 7.422911e-01 2.577086e-01  
## 30 7.051007e-15 9.958686e-01 4.131398e-03  
## 31 4.003649e-18 9.999458e-01 5.423045e-05  
## 32 9.197553e-12 9.922619e-01 7.738065e-03  
## 33 3.217886e-11 9.908705e-01 9.129531e-03  
## 34 8.498610e-17 9.991278e-01 8.722495e-04  
## 35 1.786522e-07 1.945736e-01 8.054262e-01  
## 36 7.362820e-07 9.604532e-02 9.039539e-01  
## 37 1.198049e-04 7.431685e-02 9.255633e-01  
## 38 1.280637e-03 1.730317e-03 9.969890e-01  
## 39 2.280937e-03 2.777943e-04 9.974413e-01  
## 40 5.057836e-06 3.569133e-03 9.964258e-01  
## 41 6.479848e-01 2.276832e-06 3.520130e-01  
## 42 1.690044e-05 2.020070e-01 7.979761e-01  
## 43 1.375363e-04 2.308011e-01 7.690614e-01  
## 44 8.812797e-03 1.636376e-03 9.895508e-01  
## 45 1.021910e-03 3.769988e-03 9.952081e-01  
## 46 5.540205e-05 8.524615e-03 9.914200e-01  
## 47 4.187435e-04 1.025415e-02 9.893271e-01  
## 48 7.927687e-05 1.032527e-02 9.895955e-01  
## 49 5.683882e-04 6.763850e-03 9.926678e-01  
##   
## $x  
## LD1 LD2  
## 1 -1.2483517 -1.466843072  
## 2 -1.0990064 0.825761783  
## 3 -1.6560183 0.863269086  
## 4 -2.5069104 0.323802299  
## 5 -4.6359442 1.544150677  
## 6 -5.1286767 -1.860587853  
## 7 -3.2524333 -1.992955664  
## 8 -2.9180130 -0.209110986  
## 9 -3.4062100 0.165344070  
## 10 -4.1783584 0.092761155  
## 11 -3.6303133 0.726964855  
## 12 -3.8066965 0.334971994  
## 13 -2.7900372 0.563890434  
## 14 -3.2496846 0.297464691  
## 15 -2.4091336 0.363432305  
## 16 -2.0673957 0.101313766  
## 17 -4.5689161 0.392325199  
## 18 -2.2116558 2.045986567  
## 19 -3.2891252 0.006762246  
## 20 -5.4373295 1.539843472  
## 21 -6.4268135 -0.475165945  
## 22 3.5323724 0.001528079  
## 23 4.4006485 -1.260511212  
## 24 2.9047590 -1.786685601  
## 25 4.1872986 0.118171800  
## 26 2.7403663 -0.330927076  
## 27 4.1972275 0.248107921  
## 28 2.1818426 -1.553150968  
## 29 1.5866270 -0.775656951  
## 30 4.3451984 1.395502599  
## 31 5.4571606 -0.397327273  
## 32 3.2187719 -0.720797234  
## 33 3.0229433 -1.029099283  
## 34 5.0195700 1.428455507  
## 35 1.5015278 1.605069517  
## 36 1.1790979 1.668976225  
## 37 0.3364853 -0.114806655  
## 38 -0.5702692 1.658177315  
## 39 -0.9208365 2.706527875  
## 40 0.4090110 3.314208576  
## 41 -2.5138157 2.713328569  
## 42 0.7859289 -0.207359067  
## 43 0.4720535 -1.158726416  
## 44 -0.8838698 0.935852002  
## 45 -0.4242225 1.202277745  
## 46 0.1530595 1.768205736  
## 47 -0.1412331 0.847668592  
## 48 0.1234103 1.492918393  
## 49 -0.2485265 1.021665518

Posterior : peluang masuk kelas, diambil yang terbesar di setiap observasi.

ct <- table(data13.ts$admit, pred$class)  
ct

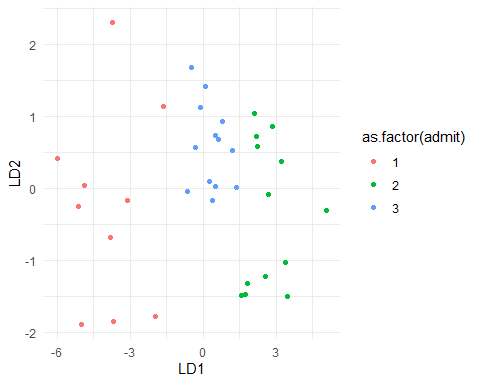
##   
## 1 2 3  
## 1 17 0 4  
## 2 0 13 2  
## 3 1 0 12

sum(diag(prop.table(ct)))

## [1] 0.8571429

Observasi yang masuk kelas 1 dan merupakan kelas 1 ada 17, sedangkan yang masuk kelas 3 dan merupakan kelas 1 ada 1 (ada 1 observasi yang salah masuk kelas). Observasi yang masuk kelas 2 dan merupakan kelas 2 ada 13. Observasi yang masuk kelas 3 dan merupakan kelas 3 ada 12, sedangkan yang masuk kelas 1 dan merupakan kelas 3 ada 4, yang masuk kelas 2 dan merupakan kelas 3 ada 2 (ada 6 observasi yang salah masuk kelas). Terlihat bahwa model berhasil mengklasifikasikan hampir semua data yaitu 85,7% dengan benar

lda.data <- cbind(data13.tr, predict(fit)$x)  
ggplot(lda.data, aes(LD1, LD2)) +  
geom\_point(aes(color = as.factor(admit))) +  
theme\_minimal()



**SPSS**

**Discriminant**

|  |  |  |
| --- | --- | --- |
| **Notes** | | |
| Output Created | | 08-DEC-2022 11:57:34 |
| Comments | |  |
| Input | Active Dataset | DataSet2 |
| Filter | <none> |
| Weight | <none> |
| Split File | <none> |
| N of Rows in Working Data File | 85 |
| Missing Value Handling | Definition of Missing | User-defined missing values are treated as missing in the analysis phase. |
| Cases Used | In the analysis phase, cases with no user- or system-missing values for any predictor variable are used. Cases with user-, system-missing, or out-of-range values for the grouping variable are always excluded. |
| Syntax | | DISCRIMINANT  /GROUPS=admit(1 3)  /VARIABLES=GPA GMAT  /ANALYSIS ALL  /METHOD=MAHAL  /PIN=.05  /POUT=.10  /PRIORS EQUAL  /HISTORY  /STATISTICS=MEAN STDDEV UNIVF BOXM COEFF RAW CROSSVALID  /PLOT=CASES  /CLASSIFY=NONMISSING POOLED. |
| Resources | Processor Time | 00:00:00,02 |
| Elapsed Time | 00:00:00,04 |

[DataSet2]

|  |  |  |  |
| --- | --- | --- | --- |
| **Analysis Case Processing Summary** | | | |
| Unweighted Cases | | N | Percent |
| Valid | | 85 | 100.0 |
| Excluded | Missing or out-of-range group codes | 0 | .0 |
| At least one missing discriminating variable | 0 | .0 |
| Both missing or out-of-range group codes and at least one missing discriminating variable | 0 | .0 |
| Total | 0 | .0 |
| Total | | 85 | 100.0 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Group Statistics** | | | | | |
| admit | | Mean | Std. Deviation | Valid N (listwise) | |
| Unweighted | Weighted |
| 1 | GPA | 3.4039 | .20871 | 31 | 31.000 |
| GMAT | 561.2258 | 67.95769 | 31 | 31.000 |
| 2 | GPA | 2.4825 | .18344 | 28 | 28.000 |
| GMAT | 447.0714 | 62.37992 | 28 | 28.000 |
| 3 | GPA | 2.9927 | .17232 | 26 | 26.000 |
| GMAT | 446.2308 | 47.40153 | 26 | 26.000 |
| Total | GPA | 2.9746 | .42900 | 85 | 85.000 |
| GMAT | 488.4471 | 81.52235 | 85 | 85.000 |

Kaus yang dianalisis ada 85 dengan 31 merupakan kelompok 1, 28 kelompok 2, dan 26 kelompok 3

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Tests of Equality of Group Means** | | | | | |
|  | Wilks' Lambda | F | df1 | df2 | Sig. |
| GPA | .191 | 173.305 | 2 | 82 | .000 |
| GMAT | .537 | 35.350 | 2 | 82 | .000 |

Karena pvalue <0.05 maka tidak ada perbedaan rata-rata yang signifikan diantara GPA dan GMAT

**Analysis 1**

**Box's Test of Equality of Covariance Matrices**

|  |  |  |
| --- | --- | --- |
| **Log Determinants** | | |
| admit | Rank | Log Determinant |
| 1 | 2 | 5.304 |
| 2 | 2 | 4.864 |
| 3 | 2 | 3.625 |
| Pooled within-groups | 2 | 4.850 |

|  |
| --- |
| The ranks and natural logarithms of determinants printed are those of the group covariance matrices. |

|  |  |  |
| --- | --- | --- |
| **Test Results** | | |
| Box's M | | 16.665 |
| F | Approx. | 2.679 |
| df1 | 6 |
| df2 | 146732.896 |
| Sig. | .013 |

|  |
| --- |
| Tests null hypothesis of equal population covariance matrices. |

**Stepwise Statistics**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Variables Entered/Removeda,b,c,d** | | | | | | | |
| Step | Entered | Min. D Squared | | | | | |
| Statistic | Between Groups | Exact F | | | |
| Statistic | df1 | df2 | Sig. |
| 1 | GPA | 4.687 | 1 and 3 | 66.283 | 1 | 82.000 | 3.665E-12 |
| 2 | GMAT | 7.434 | 2 and 3 | 49.497 | 2 | 81.000 | 9.030E-15 |

|  |
| --- |
| At each step, the variable that maximizes the Mahalanobis distance between the two closest groups is entered.a,b,c,d |
| a. Maximum number of steps is 4. |
| b. Maximum significance of F to enter is .05. |
| c. Minimum significance of F to remove is .10. |
| d. F level, tolerance, or VIN insufficient for further computation. |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Variables in the Analysis** | | | | | |
| Step | | Tolerance | Sig. of F to Remove | Min. D Squared | Between Groups |
| 1 | GPA | 1.000 | .000 |  |  |
| 2 | GPA | .969 | .000 | .000 | 2 and 3 |
| GMAT | .969 | .000 | 4.687 | 1 and 3 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Variables Not in the Analysis** | | | | | | |
| Step | | Tolerance | Min. Tolerance | Sig. of F to Enter | Min. D Squared | Between Groups |
| 0 | GPA | 1.000 | 1.000 | .000 | 4.687 | 1 and 3 |
| GMAT | 1.000 | 1.000 | .000 | .000 | 2 and 3 |
| 1 | GMAT | .969 | .969 | .000 | 7.434 | 2 and 3 |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Wilks' Lambda** | | | | | | | | |
| Step | Number of Variables | Lambda | df1 | df2 | df3 | Exact F | | |
| Statistic | df1 | df2 |
| 1 | 1 | .191 | 1 | 2 | 82 | 173.305 | 2 | 82.000 |
| 2 | 2 | .126 | 2 | 2 | 82 | 73.426 | 4 | 162.000 |

|  |  |
| --- | --- |
| **Wilks' Lambda** | |
| Step | Exact F |
| Sig. |
| 1 | .000 |
| 2 | .000 |

**Summary of Canonical Discriminant Functions**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Eigenvalues** | | | | |
| Function | Eigenvalue | % of Variance | Cumulative % | Canonical Correlation |
| 1 | 5.646a | 96.7 | 96.7 | .922 |
| 2 | .191a | 3.3 | 100.0 | .400 |

|  |
| --- |
| a. First 2 canonical discriminant functions were used in the analysis. |
| Diketahui nilai korelasi kanonik fungsi pertama sebesar 0,92, sehingga dikuadratkan menjadi 0,846 yang artinya 84,6 persen varian dari variable independent dapa dijelaskan oleh fungsi pertama diskriminan yang terbentuk. nilai korelasi kanonik fungsi kedua sebesar 0,4, sehingga dikuadratkan menjadi 0,16 yang artinya 16 persen varian dari variable independent dapa dijelaskan oleh fungsi kedua diskriminan yang terbentuk. |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Wilks' Lambda** | | | | |
| Test of Function(s) | Wilks' Lambda | Chi-square | df | Sig. |
| 1 through 2 | .126 | 168.582 | 4 | .000 |
| 2 | .840 | 14.219 | 1 | .000 |

Terdapat perbedaan yang sifinifikan antara ketiga kelompok yang didasarkan pada GPA dan GMAT pada fungsi pertama melalui fungsi kedua hal ini dapat dilihat dari signifikansi <0.05

|  |  |  |
| --- | --- | --- |
| **Standardized Canonical Discriminant Function Coefficients** | | |
|  | Function | |
| 1 | 2 |
| GPA | .951 | -.356 |
| GMAT | .518 | .874 |

|  |  |  |
| --- | --- | --- |
| **Structure Matrix** | | |
|  | Function | |
| 1 | 2 |
| GPA | .860\* | -.510 |
| GMAT | .351 | .936\* |

|  |
| --- |
| Pooled within-groups correlations between discriminating variables and standardized canonical discriminant functions  Variables ordered by absolute size of correlation within function. |
| \*. Largest absolute correlation between each variable and any discriminant function |

GPA paling membedakan ketiga kelompok.

|  |  |  |
| --- | --- | --- |
| **Canonical Discriminant Function Coefficients** | | |
|  | Function | |
| 1 | 2 |
| GPA | 5.009 | -1.877 |
| GMAT | .009 | .014 |
| (Constant) | -19.084 | -1.476 |

|  |
| --- |
| Unstandardized coefficients |

LD1 : 5.009GPA + 0.009GMAT - 19,084

LD2 = -1,877GPA + 0.014GMAT – 1,476

|  |  |  |
| --- | --- | --- |
| **Functions at Group Centroids** | | |
| admit | Function | |
| 1 | 2 |
| 1 | 2.774 | .246 |
| 2 | -2.819 | .326 |
| 3 | -.271 | -.644 |

|  |
| --- |
| Unstandardized canonical discriminant functions evaluated at group means |

**Classification Statistics**

|  |  |  |
| --- | --- | --- |
| **Classification Processing Summary** | | |
| Processed | | 85 |
| Excluded | Missing or out-of-range group codes | 0 |
| At least one missing discriminating variable | 0 |
| Used in Output | | 85 |

|  |  |  |  |
| --- | --- | --- | --- |
| **Prior Probabilities for Groups** | | | |
| admit | Prior | Cases Used in Analysis | |
| Unweighted | Weighted |
| 1 | .333 | 31 | 31.000 |
| 2 | .333 | 28 | 28.000 |
| 3 | .333 | 26 | 26.000 |
| Total | 1.000 | 85 | 85.000 |

|  |  |  |  |
| --- | --- | --- | --- |
| **Classification Function Coefficients** | | | |
|  | Admit | | |
| 1 | 2 | 3 |
| GPA | 106.250 | 78.086 | 92.670 |
| GMAT | .212 | .165 | .173 |
| (Constant) | -241.470 | -134.998 | -178.414 |

|  |
| --- |
| Fisher's linear discriminant functions |

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Casewise Statistics** | | | | | | | | | | | | |
|  | Case Number | Actual Group | Highest Group | | | | | Second Highest Group | | | Discriminant Scores | |
| Predicted Group | P(D>d | G=g) | | P(G=g | D=d) | Squared Mahalanobis Distance to Centroid | Group | P(G=g | D=d) | Squared Mahalanobis Distance to Centroid | Function 1 | Function 2 |
| p | df |
| Original | 1 | 1 | 1 | .064 | 2 | .586 | 5.490 | 3 | .409 | 6.207 | .849 | 1.582 |
| 2 | 1 | 3\*\* | .623 | 2 | .878 | .948 | 1 | .120 | 4.925 | .696 | -.534 |
| 3 | 1 | 3\*\* | .351 | 2 | .634 | 2.096 | 1 | .365 | 3.199 | 1.174 | -.554 |
| 4 | 1 | 1 | .662 | 2 | .896 | .825 | 3 | .104 | 5.129 | 1.910 | -.035 |
| 5 | 1 | 1 | .256 | 2 | .999 | 2.726 | 3 | .001 | 16.181 | 3.725 | -1.103 |
| 6 | 1 | 1 | .073 | 2 | 1.000 | 5.225 | 3 | .000 | 27.079 | 4.184 | 2.045 |
| 7 | 1 | 1 | .110 | 2 | .922 | 4.418 | 3 | .077 | 9.373 | 1.456 | 1.884 |
| 8 | 1 | 1 | .170 | 2 | .998 | 3.544 | 3 | .002 | 15.728 | 2.575 | 2.118 |
| 9 | 1 | 1 | .115 | 2 | .975 | 4.330 | 3 | .025 | 11.636 | 2.928 | -1.829 |
| 10 | 1 | 1 | .509 | 2 | 1.000 | 1.351 | 3 | .000 | 18.556 | 3.935 | .284 |
| 11 | 1 | 1 | .859 | 2 | .976 | .304 | 3 | .024 | 7.684 | 2.269 | .467 |
| 12 | 1 | 1 | .990 | 2 | .991 | .021 | 3 | .009 | 9.338 | 2.684 | .134 |
| 13 | 1 | 1 | .848 | 2 | .999 | .330 | 3 | .001 | 13.844 | 3.348 | .221 |
| 14 | 1 | 1 | .098 | 2 | 1.000 | 4.652 | 3 | .000 | 27.251 | 4.913 | -.030 |
| 15 | 1 | 1 | .082 | 2 | 1.000 | 5.014 | 3 | .000 | 26.247 | 4.075 | 2.068 |
| 16 | 1 | 1 | .820 | 2 | .992 | .397 | 3 | .008 | 9.940 | 2.870 | -.377 |
| 17 | 1 | 1 | .937 | 2 | .996 | .130 | 3 | .004 | 11.271 | 3.026 | -.011 |
| 18 | 1 | 1 | .729 | 2 | .937 | .633 | 3 | .063 | 6.021 | 2.151 | -.249 |
| 19 | 1 | 1 | .948 | 2 | .984 | .107 | 3 | .016 | 8.374 | 2.548 | .009 |
| 20 | 1 | 1 | .606 | 2 | .865 | 1.001 | 3 | .135 | 4.722 | 1.826 | -.074 |
| 21 | 1 | 1 | .463 | 2 | .765 | 1.542 | 3 | .235 | 3.907 | 1.535 | .159 |
| 22 | 1 | 1 | .636 | 2 | .999 | .905 | 3 | .001 | 15.968 | 3.680 | -.044 |
| 23 | 1 | 1 | .927 | 2 | .985 | .151 | 3 | .015 | 8.502 | 2.436 | .439 |
| 24 | 1 | 3\*\* | .099 | 2 | .523 | 4.616 | 1 | .477 | 4.803 | 1.639 | -1.629 |
| 25 | 1 | 1 | .769 | 2 | .998 | .524 | 3 | .002 | 13.350 | 3.029 | .923 |
| 26 | 1 | 1 | .212 | 2 | .999 | 3.106 | 3 | .001 | 17.269 | 2.934 | 2.001 |
| 27 | 1 | 1 | .360 | 2 | 1.000 | 2.041 | 3 | .000 | 21.153 | 4.166 | .568 |
| 28 | 1 | 1 | .982 | 2 | .989 | .037 | 3 | .011 | 9.007 | 2.585 | .278 |
| 29 | 1 | 1 | .108 | 2 | 1.000 | 4.443 | 3 | .000 | 22.131 | 4.413 | -1.079 |
| 30 | 1 | 1 | .037 | 2 | 1.000 | 6.611 | 3 | .000 | 32.952 | 5.284 | .803 |
| 31 | 1 | 3\*\* | .362 | 2 | .703 | 2.034 | 1 | .296 | 3.762 | 1.146 | -.808 |
| 32 | 2 | 2 | .955 | 2 | .947 | .093 | 3 | .053 | 5.866 | -2.540 | .202 |
| 33 | 2 | 2 | .881 | 2 | .991 | .253 | 3 | .009 | 9.563 | -3.271 | .105 |
| 34 | 2 | 2 | .325 | 2 | 1.000 | 2.248 | 3 | .000 | 17.499 | -4.004 | 1.245 |
| 35 | 2 | 2 | .351 | 2 | .992 | 2.093 | 3 | .008 | 11.786 | -2.714 | 1.768 |
| 36 | 2 | 2 | .220 | 2 | .845 | 3.025 | 3 | .155 | 6.421 | -1.568 | 1.533 |
| 37 | 2 | 2 | .563 | 2 | .997 | 1.150 | 3 | .003 | 13.091 | -3.835 | -.019 |
| 38 | 2 | 2 | .844 | 2 | .973 | .338 | 3 | .027 | 7.519 | -2.982 | -.233 |
| 39 | 2 | 2 | .968 | 2 | .962 | .065 | 3 | .038 | 6.525 | -2.588 | .432 |
| 40 | 2 | 2 | .531 | 2 | .997 | 1.268 | 3 | .003 | 13.026 | -3.845 | -.139 |
| 41 | 2 | 2 | .727 | 2 | .995 | .638 | 3 | .005 | 11.099 | -3.134 | 1.060 |
| 42 | 2 | 2 | .620 | 2 | .792 | .956 | 3 | .207 | 3.637 | -2.162 | -.399 |
| 43 | 2 | 2 | .544 | 2 | .749 | 1.218 | 3 | .251 | 3.409 | -2.113 | -.523 |
| 44 | 2 | 2 | .354 | 2 | .956 | 2.077 | 3 | .044 | 8.240 | -2.096 | 1.572 |
| 45 | 2 | 2 | .591 | 2 | .972 | 1.053 | 3 | .028 | 8.174 | -2.401 | 1.263 |
| 46 | 2 | 2 | .406 | 2 | .754 | 1.802 | 3 | .246 | 4.043 | -1.593 | .871 |
| 47 | 2 | 2 | .159 | 2 | .995 | 3.683 | 3 | .005 | 14.094 | -3.984 | -1.200 |
| 48 | 2 | 2 | .110 | 2 | 1.000 | 4.421 | 3 | .000 | 22.747 | -4.920 | .422 |
| 49 | 2 | 2 | .889 | 2 | .990 | .236 | 3 | .010 | 9.441 | -2.995 | .779 |
| 50 | 2 | 2 | .265 | 2 | .883 | 2.658 | 3 | .117 | 6.700 | -1.702 | 1.513 |
| 51 | 2 | 2 | .476 | 2 | .997 | 1.483 | 3 | .003 | 13.041 | -3.187 | 1.486 |
| 52 | 2 | 2 | .759 | 2 | .988 | .551 | 3 | .012 | 9.443 | -2.823 | 1.068 |
| 53 | 2 | 2 | .207 | 2 | 1.000 | 3.148 | 3 | .000 | 19.677 | -4.593 | .353 |
| 54 | 2 | 2 | .063 | 2 | .999 | 5.516 | 3 | .001 | 18.787 | -4.563 | -1.248 |
| 55 | 2 | 2 | .609 | 2 | .918 | .993 | 3 | .082 | 5.830 | -2.686 | -.662 |
| 56 | 2 | 2 | .341 | 2 | .891 | 2.152 | 3 | .109 | 6.350 | -1.788 | 1.368 |
| 57 | 2 | 2 | .386 | 2 | .652 | 1.905 | 3 | .348 | 3.164 | -2.042 | -.815 |
| 58 | 2 | 3\*\* | .354 | 2 | .755 | 2.078 | 2 | .245 | 4.329 | -1.545 | -1.319 |
| 59 | 2 | 3\*\* | .467 | 2 | .867 | 1.521 | 2 | .133 | 5.278 | -1.269 | -1.369 |
| 60 | 3 | 3 | .623 | 2 | .891 | .948 | 2 | .103 | 5.258 | -.526 | .295 |
| 61 | 3 | 3 | .589 | 2 | .878 | 1.057 | 2 | .116 | 5.108 | -.559 | .343 |
| 62 | 3 | 3 | .704 | 2 | .980 | .703 | 1 | .016 | 8.887 | .233 | -1.314 |
| 63 | 3 | 3 | .194 | 2 | .982 | 3.276 | 1 | .017 | 11.397 | .523 | -2.270 |
| 64 | 3 | 3 | .861 | 2 | .901 | .299 | 2 | .097 | 4.750 | -.779 | -.440 |
| 65 | 3 | 3 | .080 | 2 | .993 | 5.056 | 2 | .007 | 14.993 | -.625 | -2.865 |
| 66 | 3 | 1\*\* | .031 | 2 | .534 | 6.937 | 3 | .466 | 7.206 | 1.891 | -2.235 |
| 67 | 3 | 3 | .741 | 2 | .919 | .600 | 2 | .074 | 5.645 | -.453 | .109 |
| 68 | 3 | 3 | .584 | 2 | .709 | 1.077 | 2 | .291 | 2.856 | -1.255 | -.315 |
| 69 | 3 | 3 | .717 | 2 | .983 | .666 | 1 | .013 | 9.355 | .158 | -1.339 |
| 70 | 3 | 3 | .907 | 2 | .964 | .195 | 1 | .021 | 7.859 | .028 | -.319 |
| 71 | 3 | 3 | .488 | 2 | .750 | 1.436 | 2 | .249 | 3.642 | -.911 | .369 |
| 72 | 3 | 3 | .812 | 2 | .890 | .417 | 2 | .109 | 4.617 | -.916 | -.671 |
| 73 | 3 | 3 | .907 | 2 | .923 | .196 | 2 | .075 | 5.214 | -.687 | -.492 |
| 74 | 3 | 3 | .155 | 2 | .717 | 3.730 | 2 | .275 | 5.646 | -.632 | 1.253 |
| 75 | 3 | 3 | .382 | 2 | .512 | 1.924 | 2 | .488 | 2.020 | -1.409 | .149 |
| 76 | 3 | 3 | .737 | 2 | .931 | .611 | 1 | .066 | 5.910 | .510 | -.641 |
| 77 | 3 | 3 | .899 | 2 | .977 | .212 | 1 | .016 | 8.389 | .113 | -.898 |
| 78 | 3 | 3 | .897 | 2 | .979 | .217 | 2 | .017 | 8.313 | -.318 | -1.108 |
| 79 | 3 | 3 | .569 | 2 | .912 | 1.128 | 1 | .076 | 6.103 | .303 | .249 |
| 80 | 3 | 3 | .976 | 2 | .978 | .049 | 2 | .015 | 8.442 | -.155 | -.833 |
| 81 | 3 | 3 | .727 | 2 | .983 | .638 | 2 | .015 | 9.004 | -.389 | -1.434 |
| 82 | 3 | 3 | .671 | 2 | .870 | .798 | 2 | .127 | 4.645 | -.671 | .155 |
| 83 | 3 | 3 | .987 | 2 | .972 | .025 | 2 | .017 | 8.070 | -.126 | -.579 |
| 84 | 3 | 3 | .863 | 2 | .979 | .295 | 2 | .018 | 8.313 | -.360 | -1.180 |
| 85 | 3 | 3 | .969 | 2 | .976 | .064 | 1 | .012 | 8.860 | -.036 | -.736 |
| Cross-validatedb | 1 | 1 | 3\*\* | .046 | 2 | .507 | 6.140 | 1 | .487 | 6.221 |  |  |
| 2 | 1 | 3\*\* | .608 | 2 | .905 | .994 | 1 | .093 | 5.538 |  |  |
| 3 | 1 | 3\*\* | .342 | 2 | .665 | 2.144 | 1 | .335 | 3.516 |  |  |
| 4 | 1 | 1 | .644 | 2 | .893 | .879 | 3 | .107 | 5.119 |  |  |
| 5 | 1 | 1 | .226 | 2 | .999 | 2.978 | 3 | .001 | 16.236 |  |  |
| 6 | 1 | 1 | .052 | 2 | 1.000 | 5.900 | 3 | .000 | 28.397 |  |  |
| 7 | 1 | 1 | .085 | 2 | .899 | 4.935 | 3 | .101 | 9.305 |  |  |
| 8 | 1 | 1 | .141 | 2 | .997 | 3.913 | 3 | .003 | 15.812 |  |  |
| 9 | 1 | 1 | .089 | 2 | .967 | 4.830 | 3 | .033 | 11.609 |  |  |
| 10 | 1 | 1 | .484 | 2 | 1.000 | 1.450 | 3 | .000 | 18.636 |  |  |
| 11 | 1 | 1 | .851 | 2 | .974 | .322 | 3 | .026 | 7.604 |  |  |
| 12 | 1 | 1 | .989 | 2 | .990 | .022 | 3 | .010 | 9.226 |  |  |
| 13 | 1 | 1 | .840 | 2 | .999 | .349 | 3 | .001 | 13.728 |  |  |
| 14 | 1 | 1 | .074 | 2 | 1.000 | 5.212 | 3 | .000 | 28.495 |  |  |
| 15 | 1 | 1 | .059 | 2 | 1.000 | 5.645 | 3 | .000 | 27.419 |  |  |
| 16 | 1 | 1 | .810 | 2 | .991 | .421 | 3 | .009 | 9.819 |  |  |
| 17 | 1 | 1 | .934 | 2 | .996 | .137 | 3 | .004 | 11.139 |  |  |
| 18 | 1 | 1 | .714 | 2 | .934 | .673 | 3 | .066 | 5.984 |  |  |
| 19 | 1 | 1 | .945 | 2 | .983 | .113 | 3 | .017 | 8.280 |  |  |
| 20 | 1 | 1 | .586 | 2 | .861 | 1.069 | 3 | .139 | 4.724 |  |  |
| 21 | 1 | 1 | .436 | 2 | .757 | 1.658 | 3 | .243 | 3.927 |  |  |
| 22 | 1 | 1 | .617 | 2 | .999 | .966 | 3 | .001 | 15.920 |  |  |
| 23 | 1 | 1 | .923 | 2 | .984 | .160 | 3 | .016 | 8.405 |  |  |
| 24 | 1 | 3\*\* | .102 | 2 | .602 | 4.561 | 1 | .398 | 5.392 |  |  |
| 25 | 1 | 1 | .757 | 2 | .998 | .557 | 3 | .002 | 13.232 |  |  |
| 26 | 1 | 1 | .182 | 2 | .999 | 3.410 | 3 | .001 | 17.403 |  |  |
| 27 | 1 | 1 | .331 | 2 | 1.000 | 2.209 | 3 | .000 | 21.446 |  |  |
| 28 | 1 | 1 | .981 | 2 | .988 | .039 | 3 | .012 | 8.900 |  |  |
| 29 | 1 | 1 | .084 | 2 | 1.000 | 4.964 | 3 | .000 | 22.760 |  |  |
| 30 | 1 | 1 | .022 | 2 | 1.000 | 7.607 | 3 | .000 | 35.504 |  |  |
| 31 | 1 | 3\*\* | .355 | 2 | .740 | 2.069 | 1 | .259 | 4.166 |  |  |
| 32 | 2 | 2 | .952 | 2 | .945 | .099 | 3 | .055 | 5.802 |  |  |
| 33 | 2 | 2 | .874 | 2 | .990 | .270 | 3 | .010 | 9.464 |  |  |
| 34 | 2 | 2 | .293 | 2 | 1.000 | 2.457 | 3 | .000 | 17.773 |  |  |
| 35 | 2 | 2 | .319 | 2 | .991 | 2.284 | 3 | .009 | 11.776 |  |  |
| 36 | 2 | 2 | .188 | 2 | .819 | 3.341 | 3 | .181 | 6.356 |  |  |
| 37 | 2 | 2 | .538 | 2 | .997 | 1.240 | 3 | .003 | 13.078 |  |  |
| 38 | 2 | 2 | .835 | 2 | .972 | .361 | 3 | .028 | 7.427 |  |  |
| 39 | 2 | 2 | .966 | 2 | .960 | .069 | 3 | .040 | 6.448 |  |  |
| 40 | 2 | 2 | .504 | 2 | .997 | 1.369 | 3 | .003 | 13.017 |  |  |
| 41 | 2 | 2 | .710 | 2 | .994 | .684 | 3 | .006 | 11.021 |  |  |
| 42 | 2 | 2 | .598 | 2 | .785 | 1.028 | 3 | .215 | 3.618 |  |  |
| 43 | 2 | 2 | .518 | 2 | .739 | 1.314 | 3 | .261 | 3.392 |  |  |
| 44 | 2 | 2 | .322 | 2 | .950 | 2.267 | 3 | .050 | 8.166 |  |  |
| 45 | 2 | 2 | .567 | 2 | .970 | 1.134 | 3 | .030 | 8.084 |  |  |
| 46 | 2 | 2 | .375 | 2 | .735 | 1.960 | 3 | .265 | 4.002 |  |  |
| 47 | 2 | 2 | .128 | 2 | .994 | 4.104 | 3 | .006 | 14.272 |  |  |
| 48 | 2 | 2 | .083 | 2 | 1.000 | 4.975 | 3 | .000 | 23.758 |  |  |
| 49 | 2 | 2 | .882 | 2 | .989 | .251 | 3 | .011 | 9.342 |  |  |
| 50 | 2 | 2 | .232 | 2 | .865 | 2.922 | 3 | .135 | 6.631 |  |  |
| 51 | 2 | 2 | .448 | 2 | .997 | 1.605 | 3 | .003 | 13.042 |  |  |
| 52 | 2 | 2 | .745 | 2 | .988 | .589 | 3 | .012 | 9.348 |  |  |
| 53 | 2 | 2 | .175 | 2 | 1.000 | 3.483 | 3 | .000 | 20.208 |  |  |
| 54 | 2 | 2 | .043 | 2 | .999 | 6.300 | 3 | .001 | 19.513 |  |  |
| 55 | 2 | 2 | .586 | 2 | .913 | 1.068 | 3 | .087 | 5.760 |  |  |
| 56 | 2 | 2 | .309 | 2 | .877 | 2.350 | 3 | .123 | 6.277 |  |  |
| 57 | 2 | 2 | .355 | 2 | .631 | 2.074 | 3 | .369 | 3.143 |  |  |
| 58 | 2 | 3\*\* | .358 | 2 | .803 | 2.056 | 2 | .197 | 4.865 |  |  |
| 59 | 2 | 3\*\* | .471 | 2 | .905 | 1.504 | 2 | .095 | 6.008 |  |  |
| 60 | 3 | 3 | .599 | 2 | .884 | 1.025 | 2 | .110 | 5.199 |  |  |
| 61 | 3 | 3 | .564 | 2 | .870 | 1.145 | 2 | .123 | 5.051 |  |  |
| 62 | 3 | 3 | .684 | 2 | .979 | .758 | 1 | .018 | 8.780 |  |  |
| 63 | 3 | 3 | .161 | 2 | .978 | 3.652 | 1 | .021 | 11.328 |  |  |
| 64 | 3 | 3 | .852 | 2 | .898 | .321 | 2 | .100 | 4.710 |  |  |
| 65 | 3 | 3 | .056 | 2 | .991 | 5.772 | 2 | .008 | 15.343 |  |  |
| 66 | 3 | 1\*\* | .032 | 2 | .686 | 6.909 | 3 | .314 | 8.474 |  |  |
| 67 | 3 | 3 | .724 | 2 | .915 | .646 | 2 | .078 | 5.581 |  |  |
| 68 | 3 | 3 | .558 | 2 | .700 | 1.167 | 2 | .300 | 2.860 |  |  |
| 69 | 3 | 3 | .698 | 2 | .982 | .718 | 1 | .014 | 9.241 |  |  |
| 70 | 3 | 3 | .901 | 2 | .962 | .209 | 1 | .022 | 7.776 |  |  |
| 71 | 3 | 3 | .458 | 2 | .735 | 1.562 | 2 | .263 | 3.616 |  |  |
| 72 | 3 | 3 | .800 | 2 | .887 | .447 | 2 | .112 | 4.579 |  |  |
| 73 | 3 | 3 | .900 | 2 | .921 | .210 | 2 | .077 | 5.164 |  |  |
| 74 | 3 | 3 | .124 | 2 | .662 | 4.183 | 2 | .328 | 5.589 |  |  |
| 75 | 3 | 2\*\* | .362 | 2 | .509 | 2.035 | 3 | .491 | 2.107 |  |  |
| 76 | 3 | 3 | .720 | 2 | .928 | .657 | 1 | .068 | 5.877 |  |  |
| 77 | 3 | 3 | .893 | 2 | .975 | .227 | 1 | .017 | 8.294 |  |  |
| 78 | 3 | 3 | .890 | 2 | .978 | .233 | 2 | .018 | 8.215 |  |  |
| 79 | 3 | 3 | .543 | 2 | .906 | 1.223 | 1 | .081 | 6.054 |  |  |
| 80 | 3 | 3 | .974 | 2 | .976 | .053 | 2 | .015 | 8.343 |  |  |
| 81 | 3 | 3 | .709 | 2 | .982 | .687 | 2 | .016 | 8.910 |  |  |
| 82 | 3 | 3 | .650 | 2 | .863 | .862 | 2 | .133 | 4.601 |  |  |
| 83 | 3 | 3 | .987 | 2 | .971 | .027 | 2 | .018 | 7.973 |  |  |
| 84 | 3 | 3 | .854 | 2 | .978 | .316 | 2 | .019 | 8.215 |  |  |
| 85 | 3 | 3 | .967 | 2 | .975 | .068 | 1 | .013 | 8.756 |  |  |

|  |
| --- |
| For the original data, squared Mahalanobis distance is based on canonical functions.  For the cross-validated data, squared Mahalanobis distance is based on observations. |
| \*\*. Misclassified case |
| b. Cross validation is done only for those cases in the analysis. In cross validation, each case is classified by the functions derived from all cases other than that case. |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Classification Resultsa,c** | | | | | | |
|  |  | admit | Predicted Group Membership | | | Total |
| 1 | 2 | 3 |
| Original | Count | 1 | 27 | 0 | 4 | 31 |
| 2 | 0 | 26 | 2 | 28 |
| 3 | 1 | 0 | 25 | 26 |
| % | 1 | 87.1 | .0 | 12.9 | 100.0 |
| 2 | .0 | 92.9 | 7.1 | 100.0 |
| 3 | 3.8 | .0 | 96.2 | 100.0 |
| Cross-validatedb | Count | 1 | 26 | 0 | 5 | 31 |
| 2 | 0 | 26 | 2 | 28 |
| 3 | 1 | 1 | 24 | 26 |
| % | 1 | 83.9 | .0 | 16.1 | 100.0 |
| 2 | .0 | 92.9 | 7.1 | 100.0 |
| 3 | 3.8 | 3.8 | 92.3 | 100.0 |

|  |
| --- |
| a. 91.8% of original grouped cases correctly classified. |
| b. Cross validation is done only for those cases in the analysis. In cross validation, each case is classified by the functions derived from all cases other than that case. |
| c. 89.4% of cross-validated grouped cases correctly classified. |

**TESTING**

